Back-Paper Exam B.Math III Year (Differential Geometry) 2017

Attempt all questions. This is a CLOSED BOOK and CLOSED NOTES exam. Results proved in class, or propositions (with or without proof) from the class notes maybe used after quoting the statements of those results. Results from exercises in the notes or Pressley's book, which haven't been solved in class must be proved in full if used.)

1. Let $c : I \to \mathbb{R}^3$ be a smooth curve of unit speed. Suppose that $\mathrm{Im}(c) \subset S^2$, where $S^2 \subset \mathbb{R}^3$ is the unit sphere centred at the origin. Show that the following are equivalent:

   (i): The torsion $\tau(t)$ of c is identically zero.

   (ii): c is the arc of a circle (not necessarily a great circle)

   (iii): The curvature $k(t)$ is a constant.

   (15mks)

2. Let 0 be a regular value of a smooth function $f : \mathbb{R}^3 \to \mathbb{R}$. Let x be a point on the smooth surface $X := f^{-1}(0)$, and $c : (-\epsilon, \epsilon) \to \mathbb{R}$ a smooth curve with $c(0) = x$. If $c'(0) \notin T_x(X)$, show that the smooth function $g := f \circ c : (-\epsilon, \epsilon) \to \mathbb{R}$ is either strictly increasing or strictly decreasing in a neighbourhood of 0, and hence changes sign at $t = 0$. (15mks)

3. Let $f : \mathbb{R} \to (0, \infty)$ be a smooth function and let X be the surface obtained by revolving the profile curve $x = f(z)$ about the z- axis. Given that the mean curvature of X is identically zero, show that $f(z) = a \cosh\left(\frac{z-c}{a}\right)$ for some real numbers $c,\ a$. That is, X is a catenoid. (15mks)